

Cambridge International AS & A Level

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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- 1** Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

[illegible]

- 2** Solve the inequality $|3x - a| > 2|x + 2a|$, where a is a positive constant. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 3 (a)** Given the complex numbers $u = a + ib$ and $w = c + id$, where a , b , c and d are real, prove that $(u + w)^* = u^* + w^*$. [2]

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- (b)** Solve the equation $(z + 2 + i)^* + (2 + i)z = 0$, giving your answer in the form $x + iy$ where x and y are real. [4]

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4 Express $\frac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$ in partial fractions.

[5]

[illegible]

- 5 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - 2i| \leq 1$ and $\text{Im } z \geq 2$. [4]

- (b) Find the greatest value of $\arg z$ for points in the shaded region, giving your answer in degrees. [3]

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- 6 (a)** Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\frac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x. \quad [3]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[Turn over

- 7 The variables x and y satisfy the differential equation

$$e^{2x} \frac{dy}{dx} = 4xy^2,$$

and it is given that $y = 1$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

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8 (a) By first expanding $(\cos^2 \theta + \sin^2 \theta)^2$, show that

$$\cos^4 \theta + \sin^4 \theta \equiv 1 - \frac{1}{2} \sin^2 2\theta. \quad [3]$$

[illegible]

for $0^\circ < \theta < 180^\circ$.

[4]

[illegible]

9 The equation of a curve is $ye^{2x} - y^2e^x = 2$.

(a) Show that $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$. [4]

[illegible]

- (b) Find the exact coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

[illegible]

- 10 With respect to the origin O , the position vectors of the points A and B are given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find a vector equation for the line l through A and B . [3]

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(b) The point C lies on l and is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$.

Find the position vector of C . [2]

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- (c) Find the possible position vectors of the point P on l such that $OP = \sqrt{14}$. [5]

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11 The equation of a curve is $y = \sqrt{\tan x}$, for $0 \leq x < \frac{1}{2}\pi$.

- (a) Express $\frac{dy}{dx}$ in terms of $\tan x$, and verify that $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$. [4]

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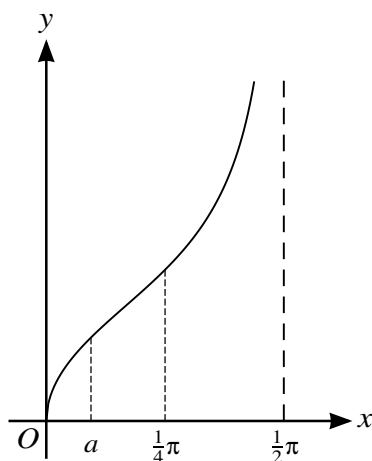
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The value of $\frac{dy}{dx}$ is also 1 at another point on the curve where $x = a$, as shown in the diagram.



- (b) Show that $t^3 + t^2 + 3t - 1 = 0$, where $t = \tan a$. [4]

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(c) Use the iterative formula

$$a_{n+1} = \tan^{-1} \left(\frac{1}{3} (1 - \tan^2 a_n - \tan^3 a_n) \right)$$

to determine a correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

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